11/11/2015 الأربعاء د. عرفة محاضرة $\boxed{6}$

Inverse Discrete Fourier Transform (IDFT)

$x(n) \xrightarrow[N]{DFT} X(K)$

$x(n)$ ⇓ Discrete time sequence

$X(K)$ ⇓ Discrete Fourier Transform

IDFT

Fourier Transform (FT)

[cont. form]

$$x(t) \xrightarrow{FT} X(j\omega) = \int_{t=-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) \xrightarrow[f^{-1}_{\text{inverse}}]{IFT} x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

f^{-1} inverse fourier transform

$$X(K) \xrightarrow[N]{IDFT} x(n)$$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi K}{N} n}$$

$$X(K) \xrightarrow[N]{IDFT} x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{-j\frac{2\pi K n}{N}}$$

notes:-

* If the discrete time sequence $x(n)$ is periodic every N sample, then the discrete Fourier transform is also periodic
* If $X(K)$ is Periodic, then the discrete time sequence $x(n)$ is also periodic.

$$X(K) \xrightarrow[N]{IDFT} x(n) = \frac{1}{N} \cdot \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi K n}{N}}$$

$x(n)$ ↓ required the discrete time seq. $x(n)$

$X(K)$ ↑ given DFT $X(K)$

$$W_N = e^{-j\frac{2\pi}{N}} \Longrightarrow \text{twiddle Factor}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_n^{-kn}$$

$$\Rightarrow x_N = \frac{1}{N} \left[W_N^{*} \right] X_N$$

$$x_N = \begin{bmatrix} x(n=0) \\ x(n=1) \\ x(n=2) \\ \vdots \\ x(N-1) \end{bmatrix} \longrightarrow \text{discrete time sequence in vector Form}$$

$$X_N = \begin{bmatrix} X(k=0) \\ X(k=1) \\ \vdots \\ X(k=N-1) \end{bmatrix} \longrightarrow \text{DFT in Vector Form}$$

$$\left[W_N^{*} \right] = \left(W_N^{-kn} \right)_{N*N}$$

for $N=3$

$$\left[W_3^{*} \right] = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \end{array} \begin{array}{c} \begin{array}{ccc} n=0 & n=1 & n=2 \end{array} \\ \begin{pmatrix} W_3^{0} & W_3^{0} & W_3^{0} \\ W_3^{0} & W_3^{-1} & W_3^{-2} \\ W_3^{0} & W_3^{-2} & W_3^{-4} \end{pmatrix} \end{array}$$

هتكون نفس W_3 المستنتجة سابقا مع عكس إشارة الجزء التخيلي

$$\left[W_3^{*} \right] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 + j\sqrt{3}/2 & -0.5 - j\sqrt{3}/2 \\ 1 & -0.5 - j\sqrt{3}/2 & -0.5 + j\sqrt{3}/2 \end{bmatrix}$$

For $\left[W_4^*\right] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$

Example :- Find IDFT of $X(k) = \{6, -2+j2, -2, -2-j2\}$ given

$$x_4 = \frac{1}{4}\left[W_4^*\right] X_4$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & 1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 6 \\ -2+j2 \\ -2 \\ -2-j2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 6-2\underbrace{-2-2}_{0}+j2-j2 \\ 6-2j-2+2+j2-2 \\ 6+2-j2-2+2+j2 \\ 6+2j+2+2-2j+2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 0 \\ 4 \\ 8 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

Properties of DFT

[1] Peridic Property

$x(n)$ is periodic then $X(K)$ is also periodic

$$X(K) = X(K+N)$$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi K n}{N}}$$

Put $K \rightarrow K+N$

$\Rightarrow$ Turn over

$$X(K+N) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi(K+N)n}{N}}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi Kn}{N}}\, e^{\frac{-j2\pi \cancel{N} n}{\cancel{N}}}$$

$$e^{-j2\pi n} = \cos(2\pi n) - j\sin(2\pi n)$$

$$= 1 \qquad n = 0, 1, 2, \ldots N-1$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi Kn}{N}} = X(K)$$

$\boxed{2}$ Linearity Property

$$x_1(n) \xrightarrow[N]{DFT} X_1(K)$$

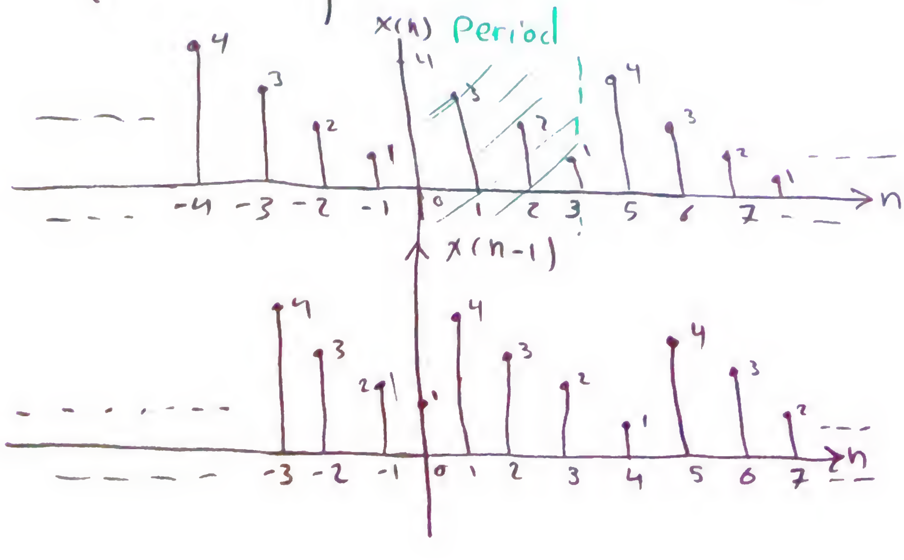
$$x_2(n) \xrightarrow[N]{DFT} X_2(K)$$

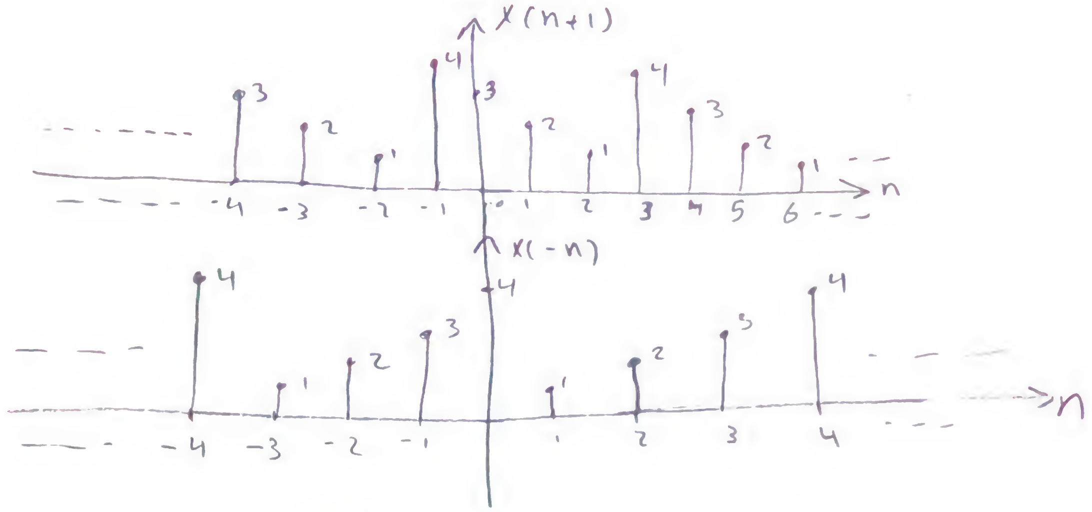
$$a_1 x_1(n) + a_2 x_2(n) \xrightarrow[N]{DFT} a_1 X_1(K) + a_2 X_2(K)$$

$\boxed{3}$ circular Shift property

If the sequence is periodic, the shift is called circular shift.

Ex: $x(n) = \{4, 3, 2, 1\}$ is periodic sequence every 4 Samples





For $\boxed{0 \leq n \leq N-1}$ one period

$x(n) \{4, 3, 2, 1\} = \{x(0), x(1), x(2), x(N-1=3)\}$

$x((n-1))_4 = \{1, 4, 3, 2\} = \{x(3), x(2), x(1), x(0)\}$

circular shift

Shift rotate to right

$x((n+1))_4 = \{3, 2, 1, 4\}$

Shift rotate to left

$x((-n)) = \{4, 1, 2, 3\}$ folding

* اعكس ما بعد أول عنصر (لنثبت الأول واعكس الباقي)

Ex: $x(n) = \{1, 0.5, 0, 1, 2\}$

Find

① $x((-n))_5 \Rightarrow \{1, 2, 1, 0, 0.5\}$

② $x((n-2))_5 \Rightarrow \{1, 2, 1, 0.5, 0\}$

③ $x((n+2))_5 \Rightarrow \{0, 1, 2, 1, 0.5\}$

delay by 2

④ $x((1-n))_5 \Rightarrow = x((-(n-1)))_5 \Rightarrow \{0.5, 1, 2, 1, 0\}$

⑤ $x((2-n))_5 \Rightarrow \{0, 0.5, 1, 2, 1\}$